

A METHOD TO FIND TRANSMISSIVITY
USING AN AQUIFER'S RESONANT FREQUENCY

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DRAFT

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A METHOD TO FIND TRANSMISSIVITY
USING AN AQUIFER'S RESONANT FREQUENCY

A Thesis

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Department of Geology

Abstract
of
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In many ways, technology is taking us to new plateaus of sensing, analyzing, and predicting behaviors in an aquifer. As a result, opportunities for better understanding the hydraulic behavior of a given well in a given aquifer are sometimes overlooked. In this study, I use the means of the “slug” used in the slug test but apply it in a cyclical fashion to create near-sinusoidal waves that are in resonance with an aquifer’s material. Hilton H. Cooper fashioned a solution [*Cooper et al.*, 1965] that uses an aquifer’s properties to evaluate waves in an aquifer that have been seismically generated. In this study, I reverse Cooper’s solution and use artificially generated waves to find the aquifer’s properties. With this technique, use is made of the slug test’s relative ease of implementation (compared to the pumping test), but the benefits are expanded by applying it cyclically. These cyclically generated resonant waves will travel further and with greater definition than a single pulse, thereby allowing testing over a greater distance and, as a consequence, the more accurate characterization of an aquifer between a stressed well and an observed well.

_____, Committee Chair
Dr. David Evans

DEDICATION

Thank you to my loving wife Melinda, whose generous support during the production of this work *seemed* limitless.

ACKNOWLEDGMENTS

I would like to acknowledge that the following work is simply the “playing out” of an idea that originally sprang from the nimble imagination of my advisor, Dr. Dave Evans. And, a special “thank you” is given to Yulia Goldshteyn for her help with the Bessel function.

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CHAPTER I

INTRODUCTION

“Today the most widespread opinion is that improving transmissivity estimates slightly by conducting additional research in well testing is useless in practice. This may be true in many cases. However, not being able to correctly understand the hydraulic behavior of a given well in a given aquifer is not a good starting point to start dealing with even more complex phenomena in practice, such as reactive solute transport or multiphase flow for CO₂ sequestration in deep aquifers” [Renard, 2005].

The apparent recent upswing in interest in the slug test is welcomed. For too long there was a growing attitude that the “variable head test” [Hvorslev, 1951] did not have sufficient accuracy, and therefore hydrogeologists should look elsewhere for more reliable methods. The slug test has long been a simple and inexpensive alternative to, or an augmentation of, the pumping test while providing comparable estimates [Spaine, 1996]. The limitation of only covering a small area around a stressed well has become an asset “as a result of the need to characterize spatial variations in K for contaminant transport investigations” [Zurbachen and Butler, 2002]. Much of the previously growing attitude was “...primarily a product of the somewhat casual approach that is often employed in slug tests” [Butler, 1996]. In 1996 the Kansas Geological Survey issued a series of guidelines for improving the accuracy of slug testing because “In order for the

potential of this technique (slug testing) to be fully realized, considerable care must be given to all phases of test design, performance, and analysis” [Butler, 1996].

In this study we use the means of the “slug” used in the slug test, but apply it in a cyclical fashion to create near-sinusoidal waves which are in resonance with an aquifer’s material. We then measure these waves as they arrive at a nearby observation well. Next, we apply the solution from *Cooper et al.* [1965] to determine the transmissivity of the aquifer through which these waves have traveled. With this technique we make use of the slug test’s relative ease of implementation (compared to the pumping test), but expand the benefits by applying it cyclically. This cyclically generated resonant wave will travel further and with greater definition than a single pulse, thereby allowing testing over a greater distance and, as a consequence, the more accurate characterization of an aquifer between a stressed well and an observed well.

CHAPTER II

PREVIOUS WORK

Hydrologists have been aware for some time that when a sudden change in volume takes place in the wellbore of a highly transmissive aquifer, the water level is restored through diminishing sinusoidal waves. This response is known as underdamped [Bredehoeft *et al.* 1966] (figure 1).

In 1965, Cooper *et al.* studied the effect of seismic waves on well water levels. Their study suggested the possibility of using wells as seismographs when it was found that the water level in a Florida well “fluctuated over a double amplitude of as much as 4.6 meters in response to the Alaska earthquake of 1964.” Cooper proposed that responding well water level fluctuations “depend not only on the characteristics of the well and aquifer but also on the period of the disturbing wave.” An analytical solution was created that related transmissivity, storativity, wave period, wellbore properties, and the wellbore amplification factor (A) (the increase of amplitude as the wave travels up the wellbore). But in their study they were only looking for relationships between pressure heads and vertical motion of land surface during seismic events. They did not capitalize on their findings by creating and testing with artificially generated cyclical waves.

Other researchers who dealt with sinusoidal waves include Ferris [1952], Carr [1969], Black & Kipp [1981], and Merritt [2004]. Merritt [2004], Ferris [1952], and Carr and Kamp [1969] were working with tidal fluctuations and other forced, long period waves rather than resonant waves and hence were concerned with amplitude attenuation

and phenomena other than resonant frequency. *Black and Kipp* [1981] created pressure waves by pumping and therefore could have varying periods. But, by using the method of pumping, the attribute of simplicity is lost due to required equipment, setup, and water storage or disposal. In addition, their solution included the calculation of volume of water pumped. Adapting their solution to a cyclical slug test would be a distraction to the goal of this paper.

Van der Kamp [1976] fashioned a solution that enabled the use of the slug test in those aquifers that produced an underdamped test result (figure 1).

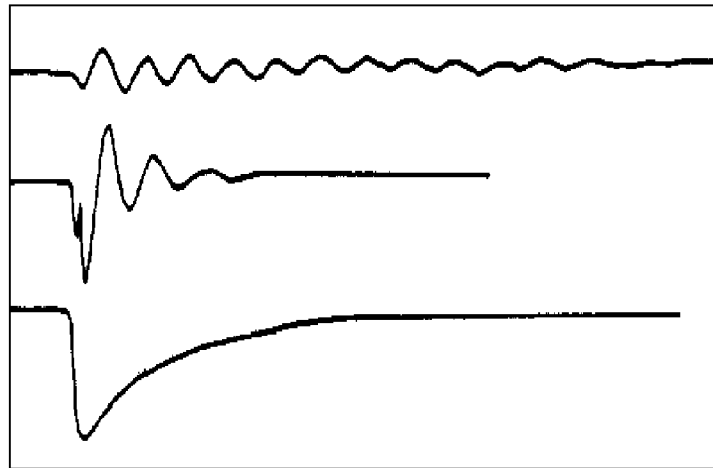


Figure 1. Figure from van der Kamp [1976] showing (from top to bottom) the underdamped, critically damped, and overdamped response.

Residual oscillatory waves were considered a hindrance to the evaluation of the results of a slug test. *Van der Kamp's* solution, though, uses the damping coefficient rather than the oscillation frequency to determine aquifer transmissivity.

No studies have been found in which oscillations were artificially created in an attempt to find the resonant frequency of an aquifer.

CHAPTER III

STUDY SITE

The study site for this research is located adjacent to, and south of, the American River and is within the on-campus wellfield of the California State University, Sacramento. The geology of the shallow subsurface is Pliocene to Recent fluvial sediments. Borings show a complex system of interbedded sands, silts, gravels, clays, and combinations of these materials. One bed that does appear homogenous, and has good hydraulic communication across the site, is at a depth of approximately 190 ft to 205 ft below the surface. This is the aquifer of interest for this study. It generally consists of medium to coarse sands, including sands from 1.5 to 2 mm in size, high resistivity, and a description in boring logs that enable it to be correlated between wells across the site. Thickness of this bed varies from 10 ft to 20 ft. Using well EX-1 (figure 2),

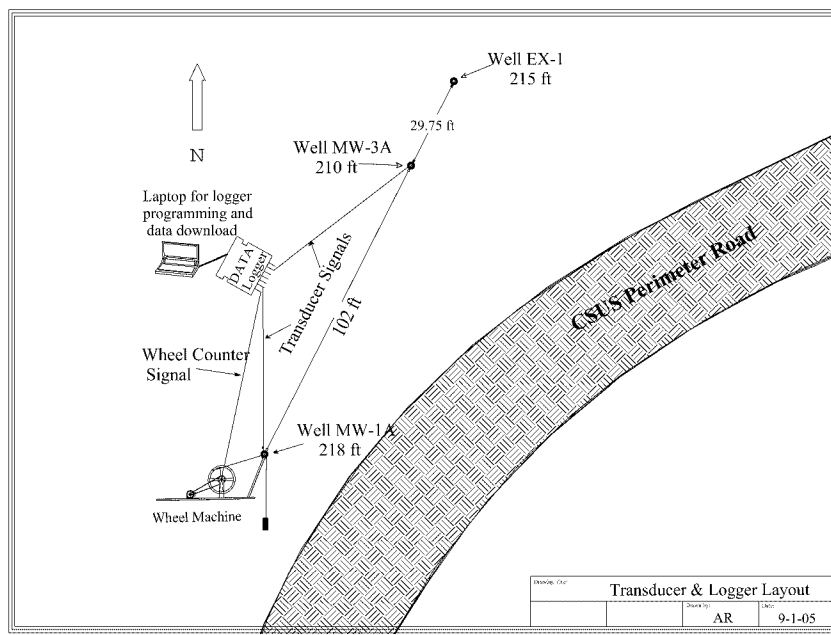


Figure 2. Local map of well location on the CSUS wellfield.

A pumping test was performed before beginning this study. The test was conducted for approximately three hours; during that time there was no evidence of leakage from an upper (groundwater) aquifer that is screened in well MW-3 from 27 ft to 47 ft. There was drawdown and associated decay at a distance of approximately 2000 ft in well DWR02-3A that is screened from 175 ft to 195 ft. This is good indication that the aquifer is a confined aquifer, is relatively homogeneous, and extends throughout the area of the wellfield. Results for transmissivity and storativity were approximately $4200 \text{ ft}^2/\text{day}$ and 0.00014 respectively. From initial measurements of hydraulic head, groundwater flow appears to be from the northeast to the southwest. Perimeter hydraulic boundaries on all sides are general head, with influx coming from a distant source to the northeast and flowing to a distant sink in the southwest. Recharge from overlying aquifers appears to be negligible; therefore, local recharge from the surface is also negligible. Outflow also exists from pumping at five on-campus irrigation wells. Impact from operation of these pumps is easily noticed in this study's test results as gradual changes in water level, and is not detrimental to the data. Continual logging of background water levels was in place for several months before testing.

The stressed well (MW-1A) and the observed well (MW-3A) are 102 ft apart, made of 5.1cm (2") PVC schedule 40 pipe and both have 15 ft screens extending down from the bottom of the confining layer (figure 3).

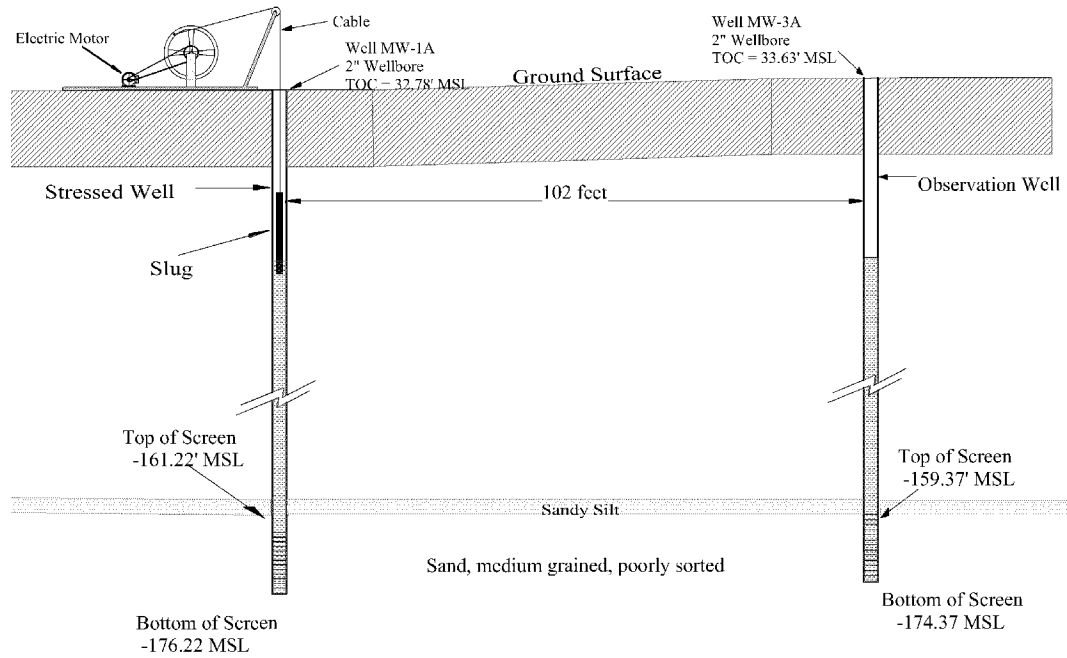


Figure 3. Diagrammatic section view of the stressed well (MW-1A) and observed well (MW-3A).

CHAPTER IV

METHODOLOGY

Determining the localized properties of an aquifer using harmonic pressure waves requires a sequence of designed steps in combination with equipment suited to these steps.

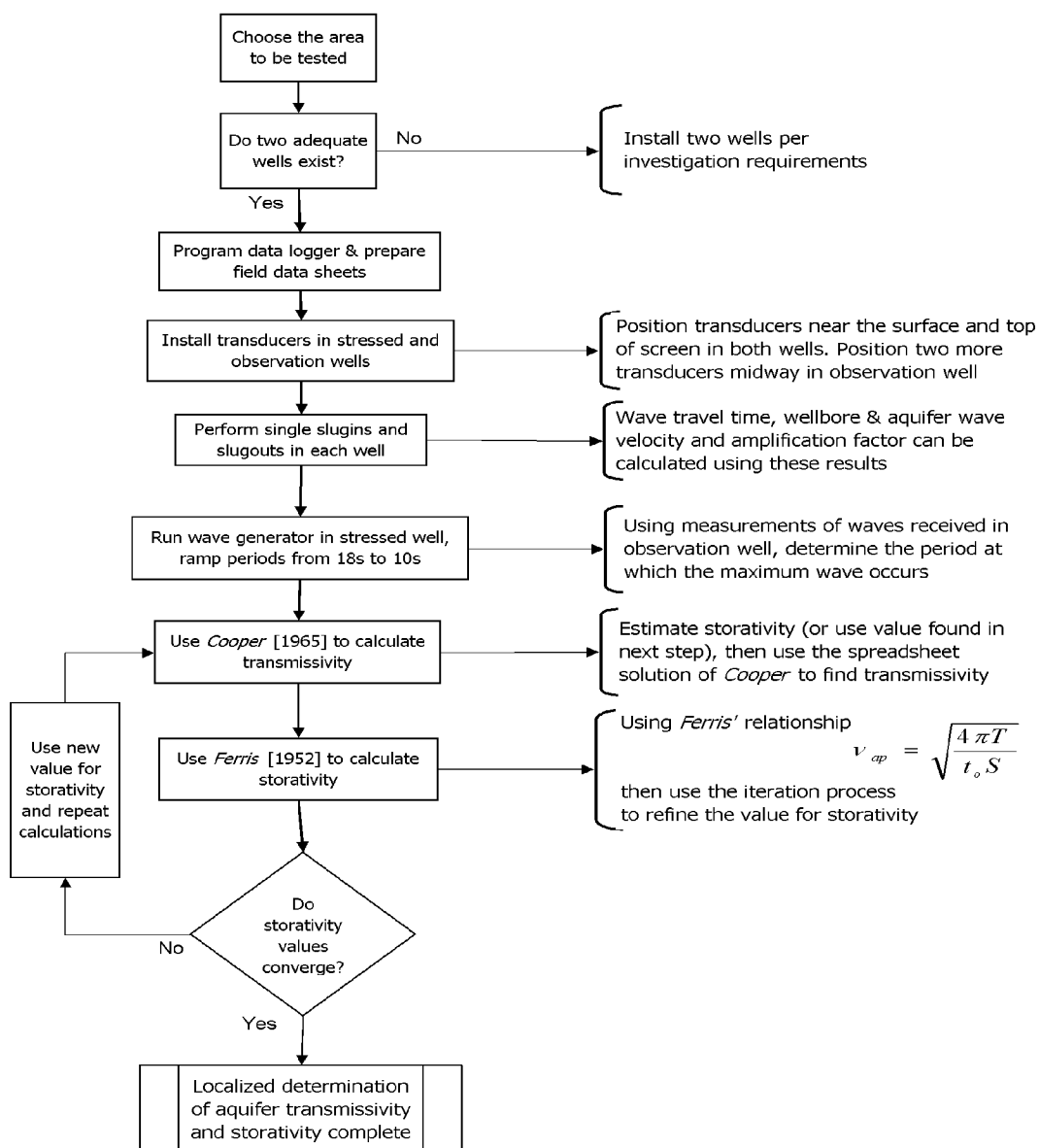


Figure 4. Flow chart of the steps in the harmonic pressure wave test.

The steps required are summarized by the flow diagram presented in figure 4 while details of the actual work carried out in this study are explained later in this chapter. Equipment required for this test and used in this study will be reviewed first.

Equipment

A convenient apparatus for creating cyclical pressure waves of varying frequency is shown in figure 5.

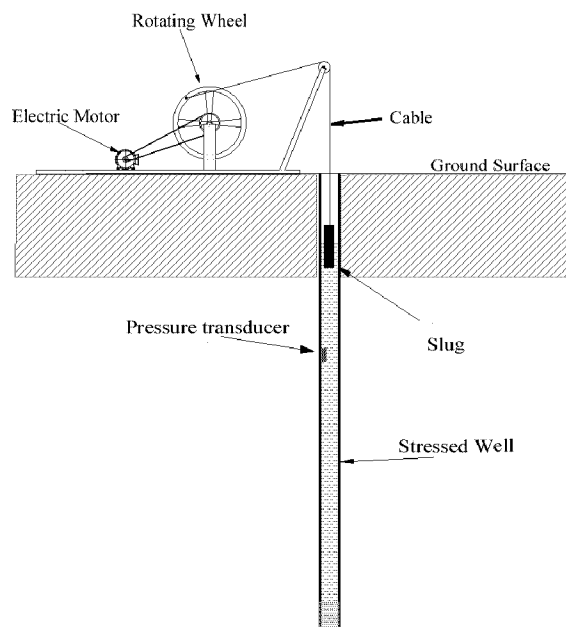


Figure 5. Generalized form of the Wave Generator.

A wheel, driven by an electric motor, lowers and raises a solid slug in the wellbore to produce a near-sinusoidal standing wave. The wheel is connected to the motor by multiple step sheaves to allow changing reduction ratios to produce frequencies over a

wide range. An electric speed controller is used to enable frequency adjustment over a finer range. A micro switch for recording each wheel rotation is installed on the large wheel and connected to the data logger for accurate time keeping. This wave generator is capable of producing wave periods from 6 s to 30 s.

Pressure transducers were used to record the arrival time and shape of the generated and residual pressure waves. Various configurations were used to allow evaluating conditions at a range of locations. The general arrangement was a pressure transducer near the surface at each well, one mid-depth in the stressed well, and one at the top of the screen in the observation well. All transducers were general duty except for that used to measure near the surface of the observation well. That transducer had a range of 0-3 psi and, in conjunction with the data logger, was capable of smoothly sensing water level variations of approximately 1/64 in. All transducers and the wave generator rotation counter were connected to a Campbell Scientifics CR1000 data logger. Per the recommendation of the KGS, [Butler, 1996] “data-acquisition equipment that enables a large quantity of high quality data to be collected should be employed,” this logger can simultaneously record signals from 8 transducers and 2 pulse counters. It can sample at up to 100 Hz, but data sampling used for this research ranged from 4 Hz to 20 Hz, depending on the level of resolution desired.

Test Procedure

Three types of tests were required to obtain the necessary data. First, slug-ins and slug-outs were performed in wells MW-1A and MW-3A, and the results recorded. From

this the necessary data were obtained to calculate the period of the residual waves, the amplification factor, and the wave velocity through the wellbore and the aquifer. Second, a series of cyclical waves were generated in MW-1A and received in well MW-3A. This series consisted of creating waves at stepped periods ranging from 30 s to 6 s in order to find the approximate resonant period. After an estimate of the resonant period was made, a third test was run which consisted of a gradual ramping of the speed of the wave generator in well MW-1A across the anticipated resonant period. Measurements were taken concurrently in observation well MW-3A that enabled identification of the maximum wave amplitude received.

In the first test, transducers were placed near the water surface and at depth in both wells MW-1A and MW-3A. Well MW-1A was slugged and recorded, and then well MW-3A was slugged and recorded. Figure 6 shows the measurements from both wells after the slug in of well MW-1A. From this graph, the overall travel time of the pressure wave from the surface of MW-1A to the surface of MW-3A (through the wellbores and aquifer) was found to be 7.8 s (± 0.2 s).

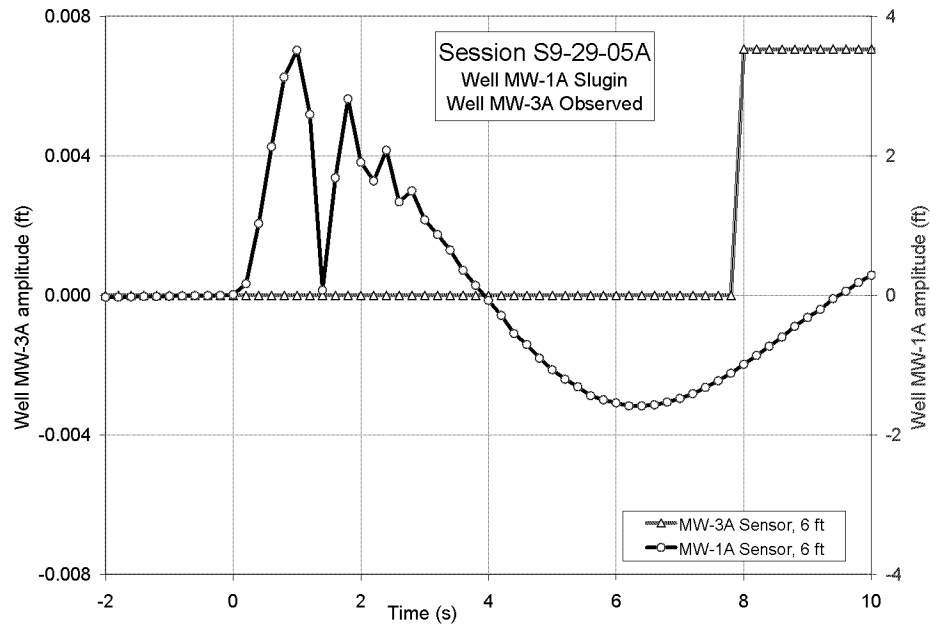


Figure 6. Test for overall travel time. Graph showing typical wave arrival at a transducer in an observation well after a slugin at a stressed well.

Figure 7 shows the results of a test to determine pressure wave velocity within a wellbore. The graph shows the time and magnitude of the wave as it passes each of the two transducers. Many variations of transducer responses were recorded due to slight deflections in transducers cables either from the slugging process or from the traveling wave, leaving some test runs questionable. This made it necessary to run many tests in order to determine the average velocity based on transducer responses that did not appear to have been disturbed by these effects. Test responses of upper and lower transducers in the same well indicated an average wave velocity in the wellbores of approximately 140 ft/s (± 10 ft/s).

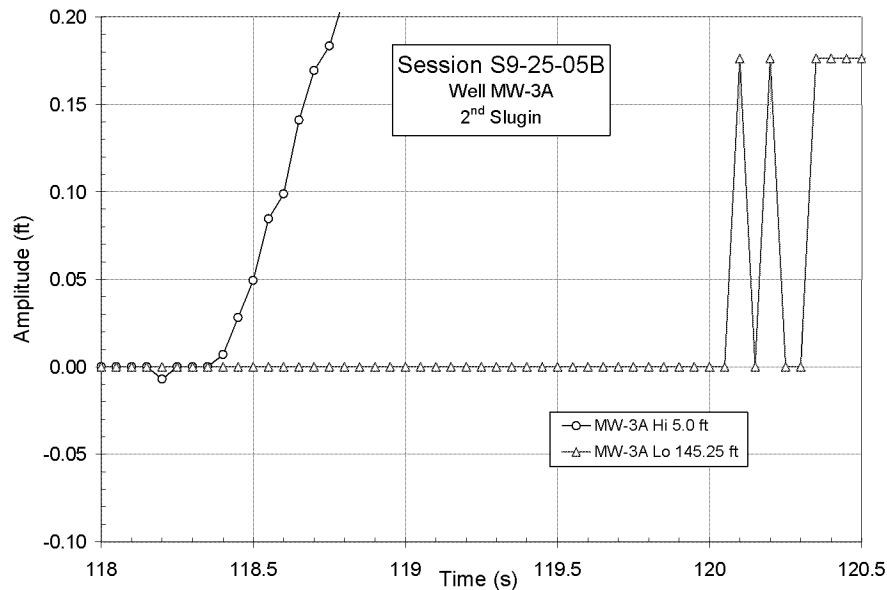


Figure 7. Test for wave velocity within a wellbore. Graph showing typical wave arrival at the upper and lower transducer within the same wellbore after a slugin at the well's water surface. This graph indicates a wave velocity of approximately 130 ft/s.

The speed of the wave in the wellbore is dependent on the physical properties of the wellbore, i.e. cables, casing material, etc. Therefore, it must be measured in-situ to be reliable. This first test made it possible to back out the travel time of the wave in the wellbore and then compute wave velocity in the aquifer at 18 ft/s (± 2 ft/s). This test also allowed measurement of the residual wave period from the underdamped response in each slugged well. MW-1A residual oscillations had a period of approximately 11.3 s and MW-3A residual oscillations had a period of approximately 11.1 s (figures 8 & 9).

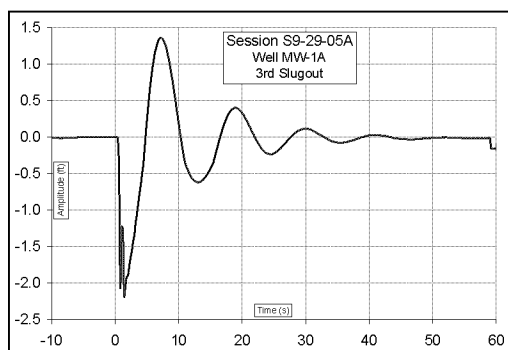


Figure 8. Graph of a typical underdamped response to a near-instantaneous withdrawal of a slug. Residual wave period is approximately 11.3 s.

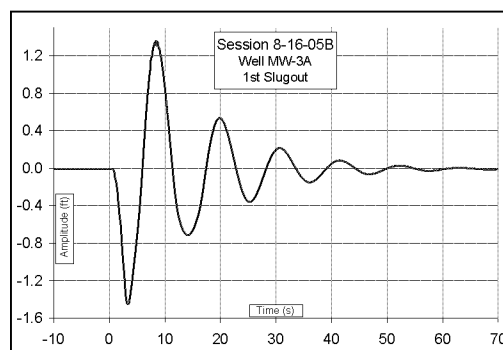


Figure 9. Similar to figure 8 except performed in well MW-3A. Residual wave period is approximately 11.1 s.

In the second test, the wave generator was set up to operate in well MW-1A. Transducers were placed near the water surface and at depth in both wells MW-1A and MW-3A. The wave generator was run at seven different sheave settings to produce wave periods of approximately 30, 20, 15, 13, 10, 8, and 6 s. This test would make known the approximate period of the aquifer resonant frequency and any wave related phenomena that might be recorded. After it was seen that the resonant period was near 14 s (figure 10), the last test was run with a sheave setting that allowed a gradual ramping of the period from approximately 16 s to 12 s. By the use of the rotational counter on the wave generator, it was possible to graph the period against the amplitude of the wave received at well MW-3A (figure 11). Matching the point of maximum amplitude from the graph of well MW-3A with the wheel period from the graph of the counter on the generator gave the resultant period of 14.06 s.

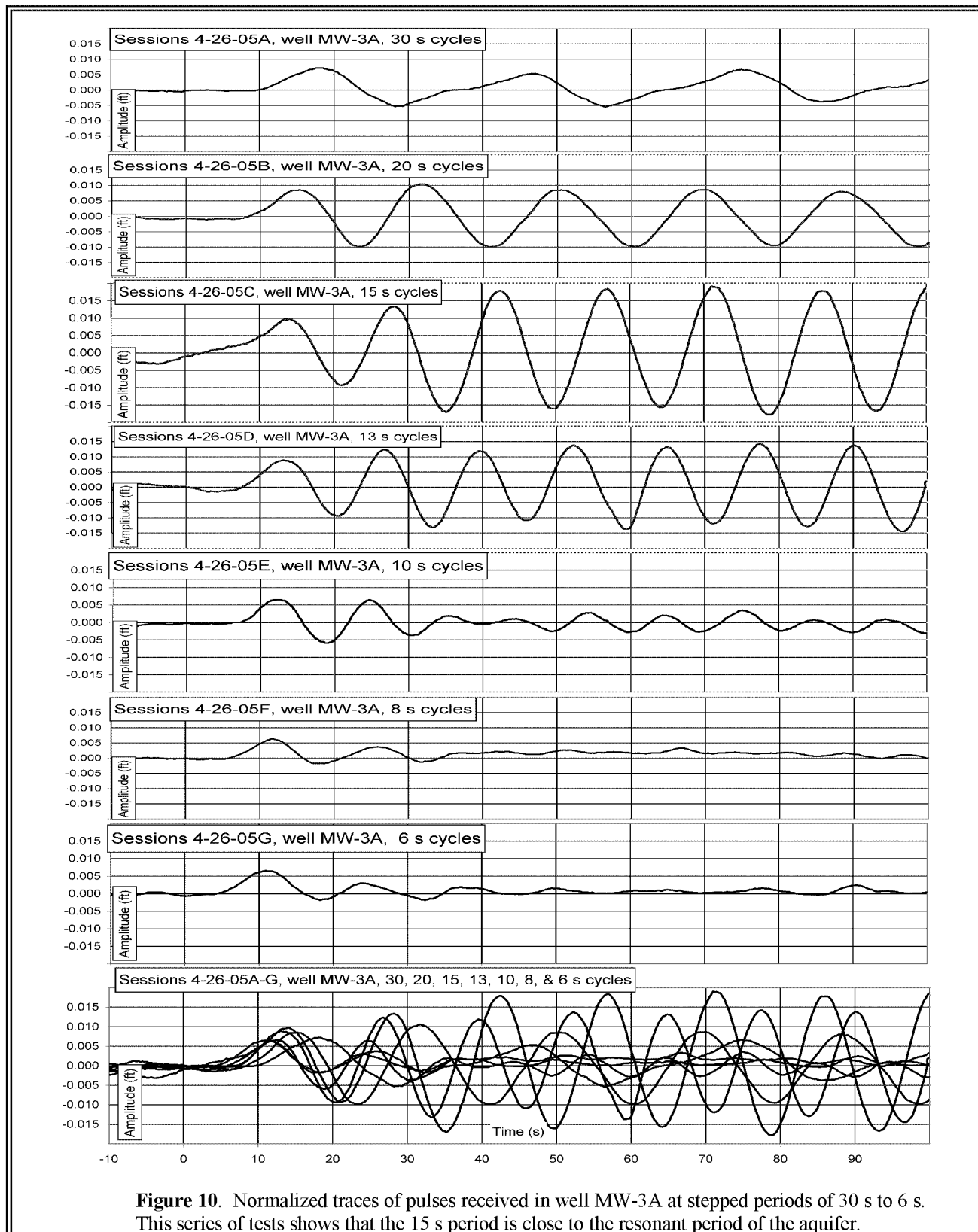


Figure 10. Normalized traces of pulses received in well MW-3A at stepped periods of 30 s to 6 s. This series of tests shows that the 15 s period is close to the resonant period of the aquifer.

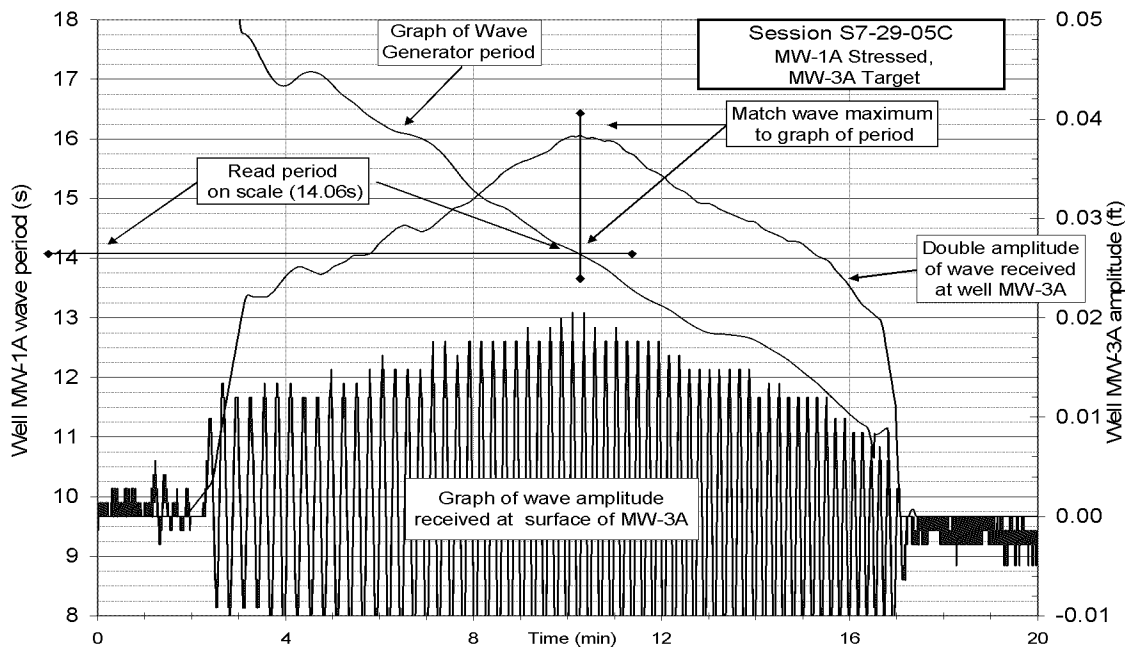


Figure 11. Resonant period determination using the point where the oscillations in the observation well (double amplitude trace) are the greatest in amplitude and then reading the wheel machine period at that time (≈ 14.06 s).

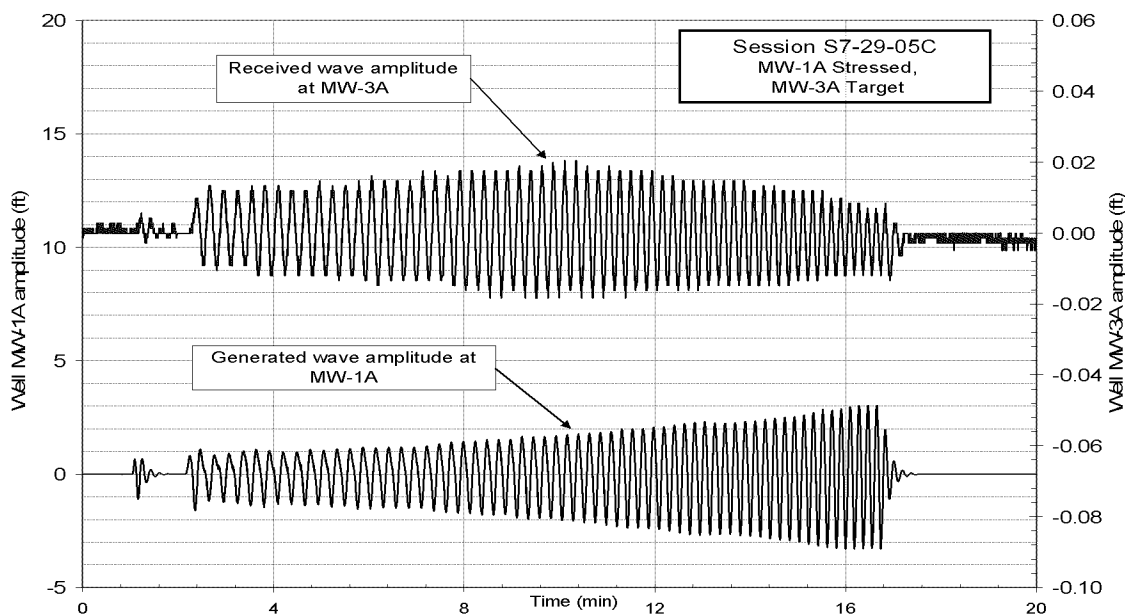


Figure 12. Resonant period wave determination showing the oscillations in observation well MW-3A (as was shown in figure 11) and showing the generated oscillations in stressed well MW-1A. Note that oscillation amplitude in the observation well decreases after passing the resonant period even though the amplitude of the generated oscillations in the stressed well continue to increase in amplitude.

CHAPTER V

ANALYSIS

By artificially generating pressure waves over a range of frequencies, it was possible to take advantage of previous research related to aquifer resonant waves. The relationships had already been derived; only a method to create the necessary standing wave was missing.

Cooper et al. [1965] offers a solution that shows a relationship between the free oscillations that result from a transient stress on an aquifer and transmissivity and *Ferris* [1952] derived a relationship between the speed of an induced pressure wave, storativity, and transmissivity. This study uses data acquired by a new procedure with their derived relationships as a way of determining the worth of using an induced pressure wave to determine aquifer transmissivity.

Basics

Each of these solutions requires an estimate of storativity. An estimate can be made by using the relationship given by *Jacob* [1940] as presented by *Cooper et al.* [1965],

$$S = \rho g d \left(\frac{n}{E_w} + \frac{b}{E_s} \right). \quad (1)$$

The drilling log for well MW-3A indicates an aquifer material of coarse sand. This puts the estimate of porosity (n) at 0.27 (0.20-0.35 per Fetter, [2001]) and bulk modulus (E_s) at 3.3×10^8 N/m² for dense sand [*Yang and Parra*, 2003]. Screen length is 15 ft, bulk

modulus (E_w) and density (d) of water is 2.2×10^9 N/m² and 1000 kg/m³. According to *Cooper et al.* [1965], in an aquifer of uncemented granular material the value of b is unity. Using these values with equation (1) provides an estimate of 0.00014 for storativity.

From the measured overall wave travel time of 7.8 s and the calculated wellbore wave velocity of 140 ft/s, the velocity of the pressure wave in the aquifer was previously calculated to be 18 ft/s. After slugging wells MW-1A and MW-3A, the residual waves had measured periods (underdamped response) of 11.3 s (S9-29-05A 3rd slugout) and 10.4 s (S8-16-05B 1st slugout) respectively (figures 8 & 9). The periods of these residual waves are controlled by the combined properties of the stressed well and the material immediately surrounding it.

Figure 10 shows the traces for each of the generated stepped wave periods (30s to 6s). The traces that show a gradual buildup of amplitude over the first several cycles are likely to be near the resonant period. This would be the periods of 13 s and 15 s. Using the approximation of a 14 s period, the wave generator was set up to stress well MW-1A over a range that included 14 s near the middle. The wave generation was started at a period of approximately 18 s and slowly ramped to 11 s. Figure 11 shows the relationship between the continual decrease in period in the stressed well and the peaking of the amplitude in the observation well at a period of 14.06 s. 14.06 s is indicated to be the period of the resonant frequency for the aquifer material and wellbore this wave has passed through.

An important comparison would be how the resonant period of 14.06 s for the wave traveling through the aquifer between MW-1A and MW-3A compares with the periods of the residual waves formed in each wellbore/aquifer system independently. As previously measured, the slug testing of wells MW-1A and MW-3A created residual waves with periods of 11.3 s and 11.1 s. Since the wave period that traveled with greatest amplitude through the aquifer material between wells MW-1A and MW-3A was 14.06 s, it makes sense that the inter-well material has a different period than either well/aquifer system. The period of the residual wave at each well is different from the period of the wave that travels best through the interlaying aquifer. That wave (14.06 s) is clearly being influenced by the properties of the aquifer in a way that residual oscillations are not. This resonant period of 14.06 s could not have been found using a single well; it could only be found by the sending of generated cyclical waves through the aquifer. Therefore, the period of 14.06 s is likely the resonant period of the aquifer at this location.

Amplification Factor

As P-waves propagate from a seismic disturbance, they generate transient pressure waves in groundwater as a result of the aquifer dilatation that takes place as the P-wave passes [*Blanchard and Byerly*, 1935]. As mentioned earlier, *Cooper et al.* [1965] observed that these pressure waves produced significantly amplified displacements in observation wells including the instance of a Florida well's induced 15 ft fluctuations in response to a distant seismic event. The water level in a well will oscillate when its

properties are in harmony with one or more components of a transient pressure wave.

Cooper *et al.*, [1965] established the analytical relationship

$$A = \left[\left(1 - \frac{\pi r_w^2}{T\tau} \text{Kei} \alpha_w - \frac{4\pi^2 H_e}{r^2 g} \right)^2 + \left(\frac{\pi r_w^2}{T\tau} \text{Ker} \alpha_w \right)^2 \right]^{-\frac{1}{2}}, \quad (2)$$

where

$$\alpha_w = r_w \left(\frac{\omega S}{T} \right)^{\frac{1}{2}}, \quad (3)$$

between a well's amplification factor (A) and aquifer transmissivity (T), storativity (S), the physical properties of a wellbore, and the frequency of a seismic wave (ω). For the known conditions at well MW-3A and solving (2) for A , figure 13 shows the relationship between A and T .

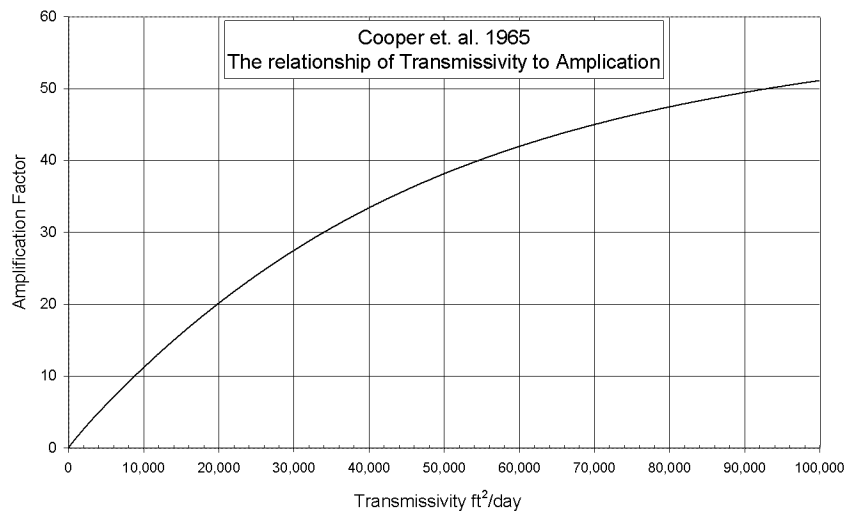


Figure 13. The relationship between transmissivity and the response of the water level in a wellbore (A) to a transient pressure wave as calculated from equation 2, and using a period of 14.06 s.

Holding T constant, (2) gives a relationship between A and wave period (τ) shown in figure 14.

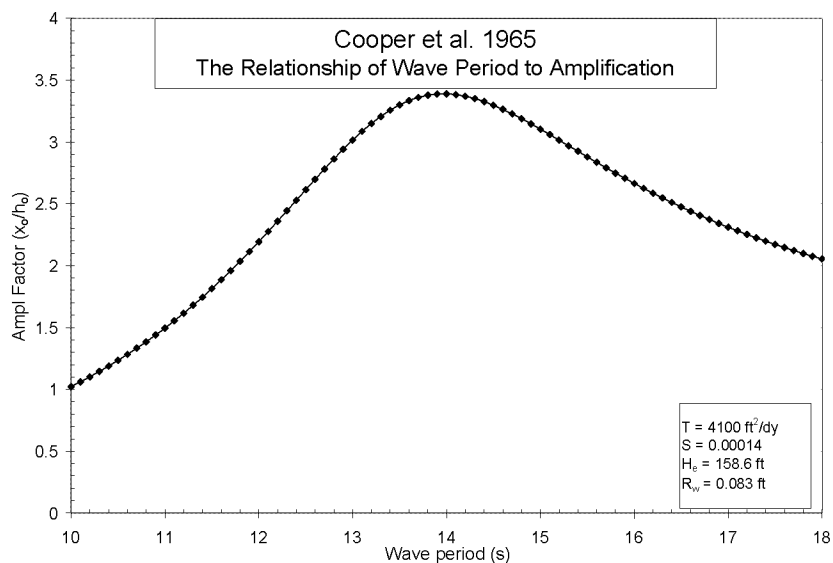


Figure 14. The relationship between wave period and the response of the water level in a wellbore to a transient pressure wave as calculated from equation 2.

By artificially generating pressure waves rather than waiting for an earthquake, the value of A can be calculated and then used to solve (2) for transmissivity.

One additional field test was set up to inject a slug of water at the bottom of well MW-3A. This was done to determine the best way to calculate the amplification factor. For this test, five transducers were installed to record the wave as it traveled up the wellbore. For injecting the water, a pump was connected to a $\frac{1}{2}$ in diameter PVC pipe that extended from ground level to just below the top of the screen. The outlet on this injection pipe was fitted with a back pressure valve that only allowed the release of water at a preset pressure. This valve was installed to assure the near instantaneous release that

is required for a proper slug [Butler, 1996]. The outlet valve was located at 205 ft below TOC, the top of the screen was 193 ft below TOC, and the sensors were at placed at 185, 143, 105, 76, and 45 ft below TOC. The fourth transducer is not shown due to a malfunction that resulted in unreliable readings.

The result of this bottom injection test is shown in figure 15.

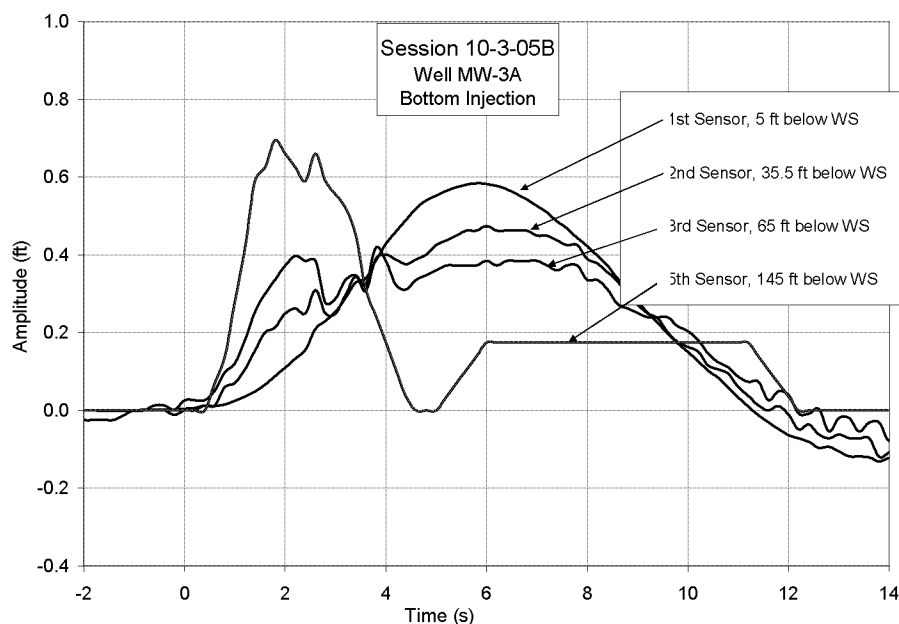


Figure 15. Graph showing a slugin near the bottom of well MW-3A and the increase in amplitude as the pressure wave travels up the wellbore. It is assumed most of the spike in the bottom transducer was a result of vibration of the injection pipe.

The initial bump in all traces is likely caused in part by movement of the injection pipe as it underwent pressurization. The three upper transducers clearly reveal the amplification of the wave as it travels toward the surface.

To determine the value of A , data was used from single slugins and slugouts such as that shown in figures 16.

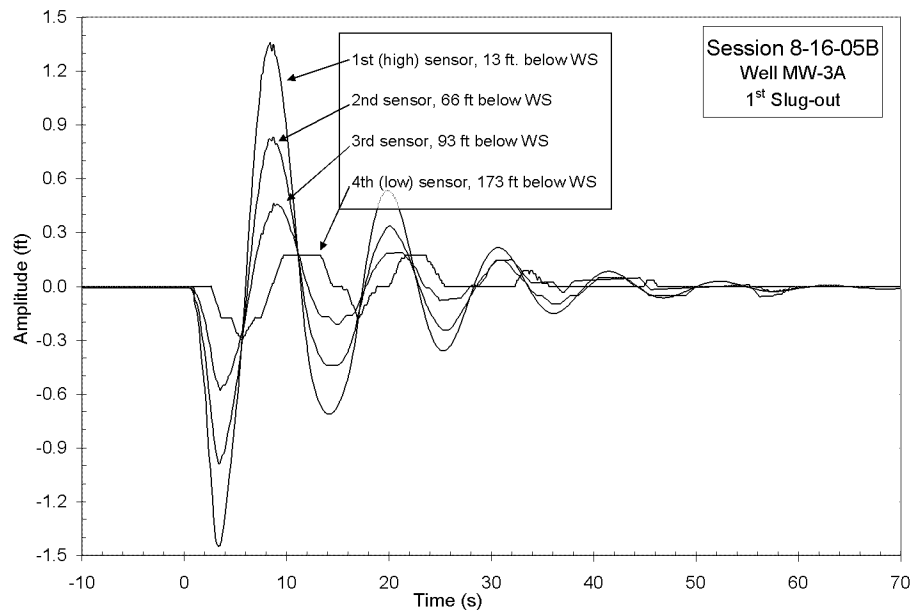


Figure 16. Slugout in a wellbore equipped with four transducers. This graph shows the period of the residual waves (11.3 s), and the increase in amplitude as the returning wave travels up the wellbore.

Only amplitudes of the waves entering the wellbore were used. To reduce the possible impact of wellbore friction, only the second and subsequent waves were used. After determining values from several tests and averaging them, the amplification factor was estimated to be 3.75. Calculation of effective height (H_e) from physical measurements of the well gives 158.63 ft, the well radius (r_w) is 0.083 ft, S was estimated earlier to be 0.00014, and the resonant period (τ) was found to be 14.06 s, which makes the angular velocity (ω) 0.4468 /s.

Using these values and solving (2) for T gives a transmissivity of 3000 ft²/dy, which is reasonably close to the 4200 ft²/dy determined by the previous pumping test.

Ferris, 1952

Ferris derived two solutions [Ferris, 1952] for aquifer transmissivity. One solution (stage-ratio method) was presented as using periodic fluctuations, but it really had more to do with amplitude attenuation of the head-change induced pressure wave.

Additionally, the fluctuations being observed were long period such as tidal fluctuations and therefore beyond the likelihood of containing any useful resonant components. The other solution (time-lag method) used the speed of the maximum or minimum of an aquifer pressure wave that results from a periodic water level change in a nearby surface water body. This solution included the relationship given by

$$v_{ap} = \frac{x}{t_1} = \sqrt{\frac{4\pi T}{t_o S}} \quad (4)$$

Using the wave speed found earlier of 18 ft/s, the estimate of S of 0.00014, and solving for T , gives the value of 5022 ft²/dy which is reasonably close to the 4200 ft²/dy determined by the previous pumping test.

By considering Ferris, 1952 in this study, we now have two equations (Cooper, 1965 and Ferris, 1952) and two unknowns (S and T). By taking the value for T found using (2) and using it to solve (4) for S , then using the iteration process, we can refine the original estimated value for storativity. This process must be repeated until there is a convergence of the storativity value. Four iterations using the current data provided the values in the following figure.

Iteration Run Number:	S used in Copper Calcs	T (ft ² /d) Calculated in Cooper Calcs	S calculated in Ferris Calcs
Run 1	0.00014	3003	0.000096
Run 2	0.000096	3049	0.000097
Run 3	0.000097	3040	0.000097

Figure 17. The iteration process using the *Cooper*, 1965 and *Ferris*, 1952 solutions results in a storativity value of 0.000097

Additional Observations

Several other observations seem relevant to an understanding of this work, and will be mentioned here.

Assumptions are:

- Those used by *Cooper et al.* [1965]:
 - Darcy's Law applies.
 - Frictional losses through the screen are negligible.
 - Water flow in well casing is everywhere vertical and uniform.
 - Flow is symmetrical about the center of the screen.
 - Wellbore friction is negligible in relation to the drawdown in the aquifer
- Frictional forces that contribute to damping are not impacting test results.
- The flow/acceleration of water past the transducers is not impacting final results.
- Well skin effects are not impacting results.
- The aquifer is homogeneous between the stressed well and the observation well.

Butler [2002] presents an argument that wellbore friction is an important consideration in wells of 5 cm diameter and smaller when performing a slug test. It is likely that wellbore friction is not a factor when working with wave frequencies. Friction in the wellbore may cause a slight phase shift in the arriving pulses, but since each cycle is impacted alike, the frequency should not be impacted. Wellbore friction may also cause attenuation in the amplitude of waves traveling within the wellbore, but for determining A , this impact should be proportional over the length of the wellbore as is the amplification factor itself.

Figure 18 was created using a value of $S=0.00014$ to provide a convenient 3-D view of the relationship between three of the variables; T , A , and τ .

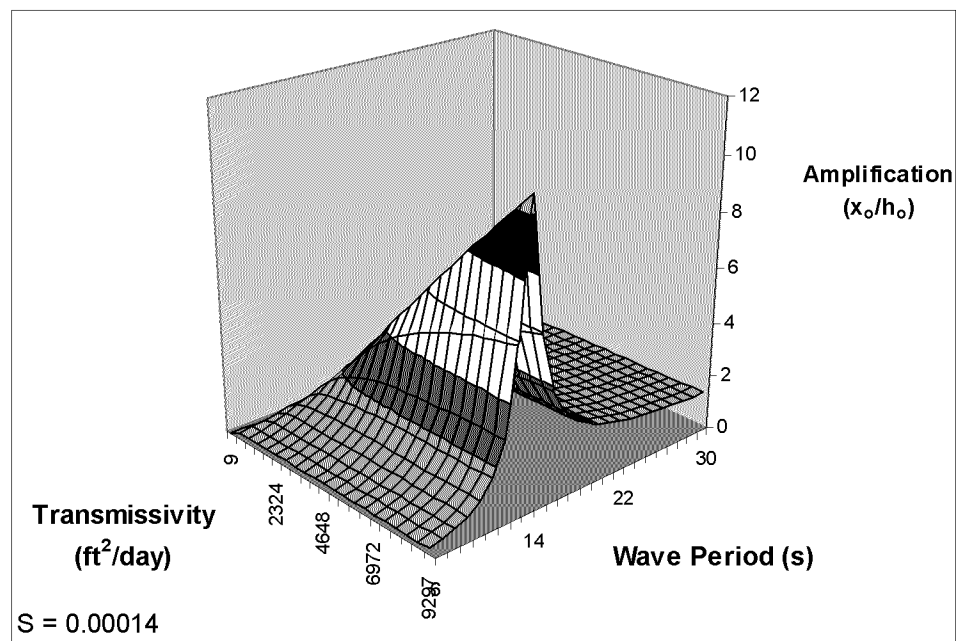


Figure 18. A generalized view of the relationship between transmissivity, wave period, and the amplification in a wellbore using a storativity value of 0.00014, as calculated from equation 2.

It is notable how sensitive the amplification factor is to wave period. Two additional 3-D graphs are presented here with different values of storativity (figures 19 and 20) to allow inspection of the impact changing this parameter has on the other constraints, and figure 21 show the relationship between T and S when using variables from the local study site.

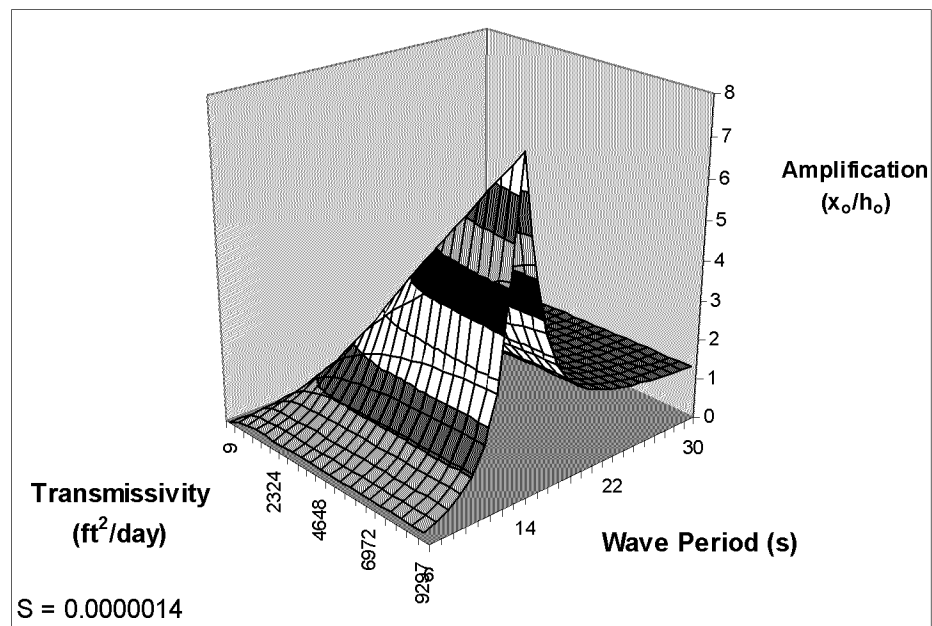


Figure 19. A generalized 3-D view of the relationship between transmissivity, wave period, and the amplification in a wellbore using a storativity value of 0.0000014, as calculated from equation 2.

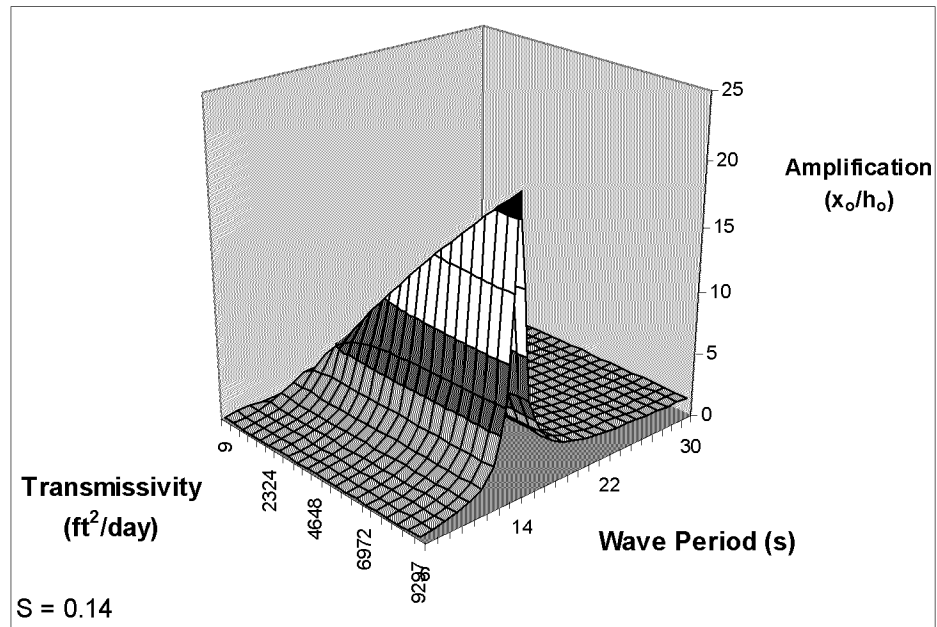


Figure 20. A generalized view of the relationship between transmissivity, wave period, and the amplification in a wellbore using a storativity value of 0.14, as calculated from equation 2.

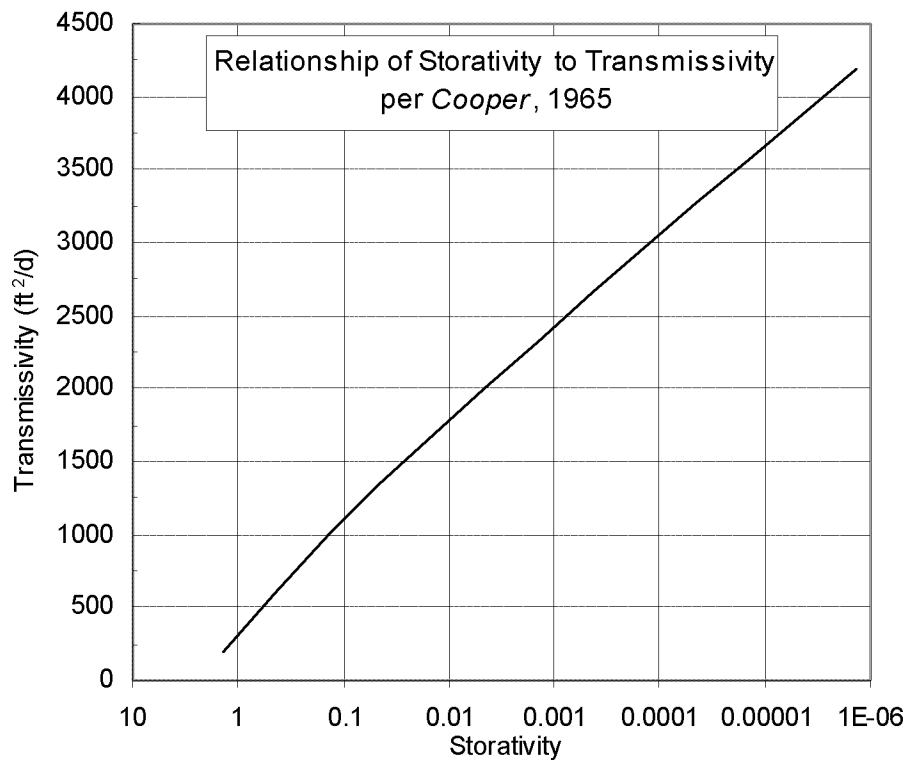


Figure 21. The relationship of storativity using equation (2), $\tau = 14.06s$, $A = 3.75$, $r_w = 0.083$ ft, and $H_c = 159.63$ ft

Additional observations can be made from figure 22 and figure 23. In figure 22, graphs from each of the listed wave periods were combined and normalized for start times and water levels. The first slug in of the cycles shows that each increase in velocity of the slug travel into the water causes a larger initial amplitude in the wave. This could be anticipated, but then the waves that were in the mid range in amplitude on the first cycle successively build and overtake the other waves. Clearly, this is the effect of the well/aquifer harmonic properties. A second observation is the overlain traces of 20 s

periods from two different wave generation sessions. Their near identical traces are an indication of the precision of field methods and equipment employed in this study.

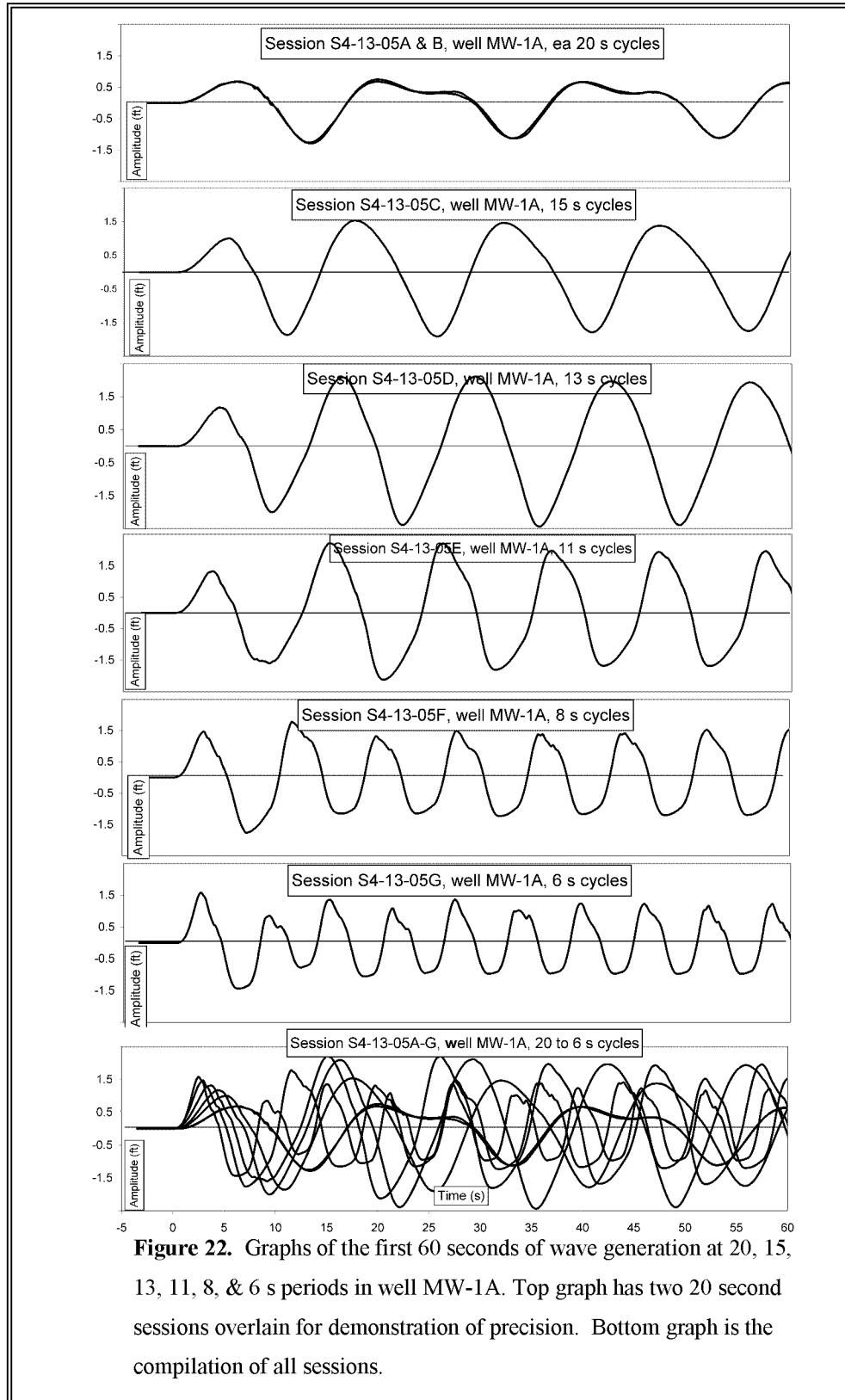
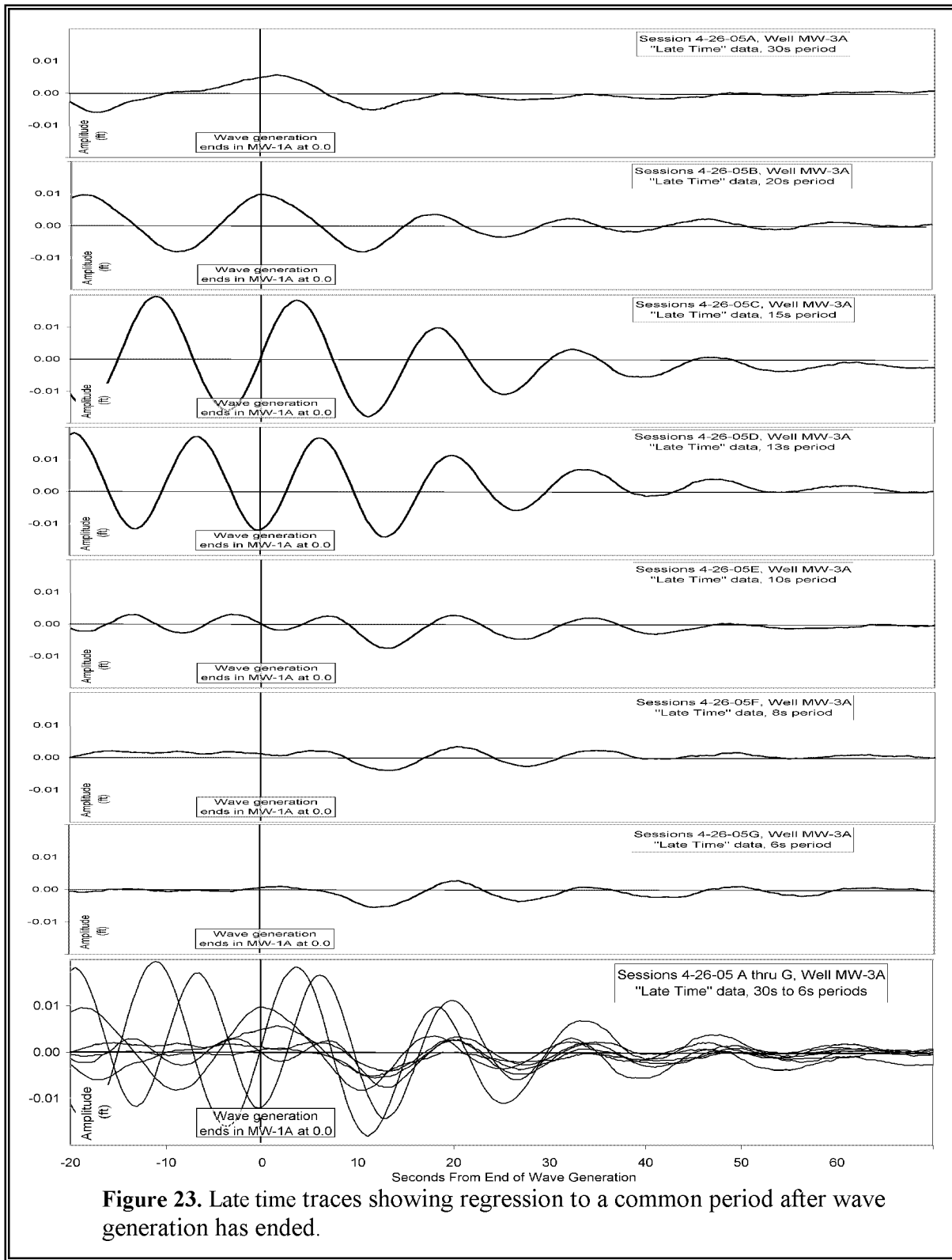


Figure 22. Graphs of the first 60 seconds of wave generation at 20, 15, 13, 11, 8, & 6 s periods in well MW-1A. Top graph has two 20 second sessions overlain for demonstration of precision. Bottom graph is the compilation of all sessions.

In figure 23, graphs from each of the listed wave periods were combined and normalized for time at the end of wave generation and water levels. The trace after the wave generation has ended shows all oscillations regressing a common period regardless of the period used to create them. Clearly, this is the effect of the well/aquifer harmonic properties. This figure also shows that the traces of the shorter period waves have been cancelled by conflicting components during slugging, but, once the slugging ends, they reappear at the same common period.



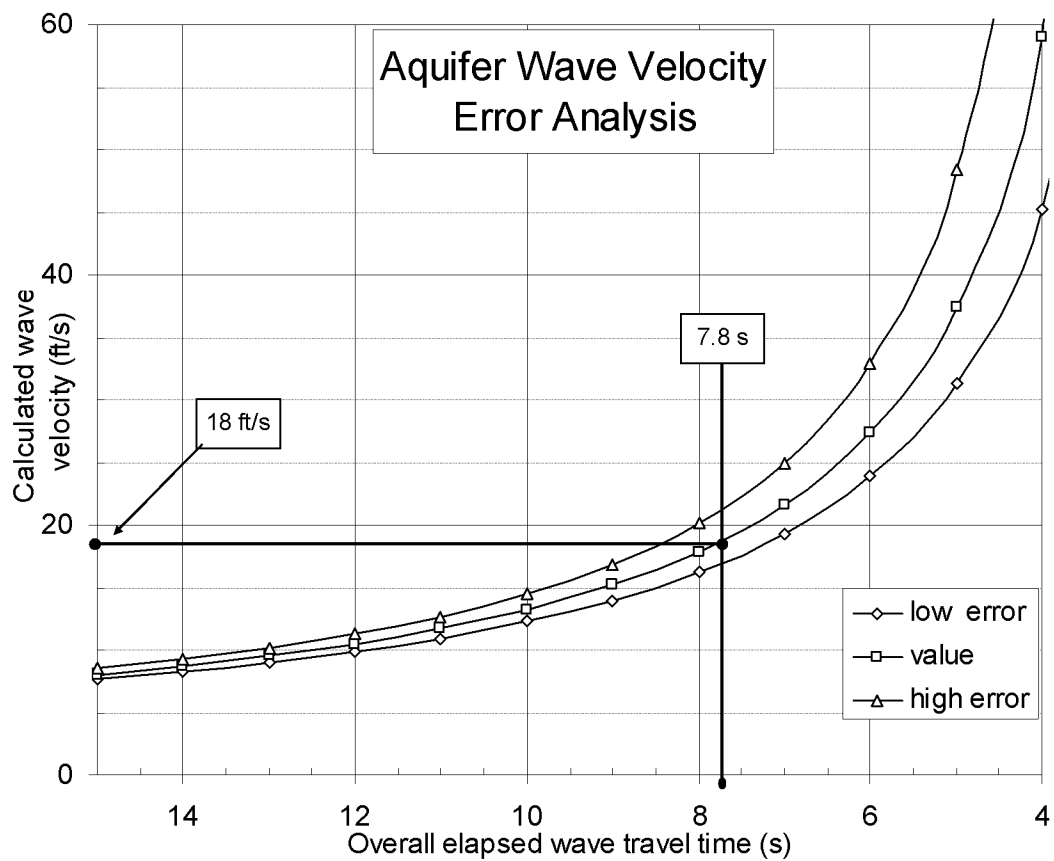


Figure 24. Wave velocity error analysis based on ± 1 ft for all linear dimensions, ± 10 ft/s for the calculated wellbore wave velocity, and ± 0.2 s for measured wave travel time between the high transducers of wells MW-1A and MW-3A.

CHAPTER VI

SUMMARY/CONCLUSION

Cooper derived the relationship between a naturally occurring wave frequency (from a seismic event) and an aquifer/wellbore's properties, but never used this relationship in the evaluation of aquifer properties. This current research shows it is possible to determine this "natural frequency" by sending artificially generated hydraulic waves of differing frequencies through an aquifer. Once known, this natural frequency can be used to calculate transmissivity using the solution that Cooper had originally intended for evaluating seismic events.

"Generally, the observation well must be fairly close (about 10 m or less) to the slugged well to be effective" [McElwee *et al.* 1995]. By using cyclical slugs, it has been demonstrated that testing can be accomplished across 102 ft with very clear definition, much further than the distance to which single slugs (as mentioned in the above statement) are commonly limited.

This test works well in high T aquifers, thereby addressing concerns expressed by *Kruseman and de Ridder* [1991] when they suggested it might not be appropriate to use a slug test in aquifers with T higher than 2,690 ft²/day. By using this test, it also eliminates the need to apply the transducer correction presented by *Zurbachen et. al.* [2002] that is otherwise necessary in a high T aquifer when using the slug test.

This new test increases the usefulness of the slug-type test by increasing its area of coverage. In addition, by the use of carefully placed observation wells, it can also help in

groundwater contamination investigations by evaluating localized site features such as high conductivity zones; it can achieve this while maintaining the advantage of simplicity over the pumping test.

APPENDIX

Nomenclature:

$A = x_o/h_o$,	amplification of pressure-head fluctuation (dimensionless)
b ,	effective proportion of aquifer area that responds elastically (dimensionless)
d ,	length of screen (L)
E_s ,	bulk modulus of aquifer (M/LT ²)
E_w ,	bulk modulus of water (M/LT ²)
g ,	gravity (L/T ²)
h_o ,	amplitude of pressure head fluctuation at screen (L)
H ,	length of water column above screen (L)
$H_e = H + 3d/8$,	effective height of water column (L)
Kei Ker,	parts of the Kelvin function (dimensionless)
n ,	porosity of aquifer (dimensionless)
r_w ,	radius of well (L)
s ,	second (T)
S ,	storativity (dimensionless)
t_l ,	lag time for pressure wave (T)
t_o ,	period of pressure wave (T)
T ,	transmissivity (L ² /T)
v_{ap} ,	apparent velocity of pressure wave (L/T)
x ,	distance of pressure wave travel (L)
x_o ,	amplitude of water level fluctuation (L)
WS,	water surface in wellbore
$\alpha_w = r_w(\omega S/T)^{1/2}$,	(dimensionless)
ω ,	angular frequency of wave (/T)
ρ ,	density of water (M/L ³)
τ ,	period of wave (T)

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